QUIZ 13 SOLUTIONS: LESSON 17
OCTOBER 10, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] How much money should you invest today at an annual interest rate of 5.5% compounded continuously so that, starting **two** years from now, you can make annual withdrawals of **$3500** in perpetuity? Round your answer to the nearest cent.

<u>**Solution**</u>: Continuously compounded interest is given by the formula

$$A = Pe^{rt}$$

where A is the final amount, P is the initial investment, r is the rate converted to a decimal, and t is time in years. Here, $r = .055$.

The goal is that every year, beginning two years from now, we can withdraw $3500. We need to determine the amount we must invest so that this is possible. We first ask: how much do we need to invest so that after 2 years we have $3500? We write

$$3500 = P_2 e^{2 \cdot .055}$$

where P_2 is the amount we invest now. Solving for P_2,

$$P_2 = \frac{3500}{e^{.055 \cdot 2}} = 3500e^{-.055 \cdot 2}.$$

To have $3500 after 3 years, we need to invest P_3 where

$$3500 = P_3 e^{.055 \cdot 3} \quad \Rightarrow \quad P_3 = \frac{3500}{e^{.055 \cdot 3}} = 3500e^{-.055 \cdot 3}.$$

To have $3500 after n years, we need to invest P_n where

$$3500 = P_n e^{.055 \cdot n} \quad \Rightarrow \quad P_n = \frac{3500}{e^{.055 \cdot n}} = 3500e^{-.055 \cdot n}.$$

Now, to determine the total amount we need to invest so that we can withdraw $3500 **every** year starting 2 years from now, we add all of our P_n together. We write

$$\sum_{n=2}^{\infty} P_n = \sum_{n=2}^{\infty} 3500e^{-.055 \cdot n}$$

$$= \sum_{n=2}^{\infty} 3500(e^{-.055})^n$$

$$= \sum_{n=0}^{\infty} 3500(e^{-.055})^{n+2}$$

$$= \sum_{n=0}^{\infty} 3500(e^{-.055})^{2}(e^{-.055})^{n}$$

$$= \frac{3500(e^{-.055})^2}{1 - e^{-.055}}$$

$$\approx \boxed{\$\ 58{,}589.71}$$